Class X Session 2023-24
Subject - Mathematics (Basic)
Sample Question Paper - 10

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A, B, C, D and E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with sub- parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

Section A

1. $(1 + \sqrt{2}) + (1 - \sqrt{2})$ is **[1]**

   a) a rational number
   b) a non-terminating decimal
   c) None of these
   d) an irrational number

2. If p_1 and p_2 are two odd prime numbers such that $p_1 > p_2$, then $p_1^2 - p_2^2$ is **[1]**

   a) an even number
   b) an odd prime number
   c) an odd number
   d) a prime number

3. If the equation $x^2 - kx + 1 = 0$ has no real roots then **[1]**

   a) $-2 < k < 2$
   b) None of these
   c) $k < -2$
   d) $k > 2$

4. The sum of the numerator and denominator of a fraction is 18. If the denominator is increased by 2, the fraction reduces to $\frac{1}{3}$. The fraction is **[1]**

   a) $\frac{-7}{11}$
   b) $\frac{5}{13}$
   c) $\frac{-5}{13}$
   d) $\frac{7}{11}$

5. If y = 1 is a common root of the equations $ay^2 + ay + 3 = 0$ and $y^2 + y + b = 0$, then ab equals **[1]**

   a) 3
   b) -3

c) 6 d) -7/2

6. AOBC is a rectangle whose three vertices are A(0, 3), O(0, 0) and B(5, 0). The length of its diagonal is **[1]**

a) 5 b) 3

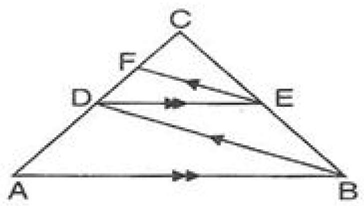
c) $\sqrt{34}$ d) 4

7. ABCD is a trapezium such that BC || AD and AB = 4 cm. If the diagonals AC and BD intersect at O such that $\frac{AO}{OC} = \frac{DO}{OB} = \frac{1}{2}$ then, Find DC. **[1]**

a) 8 cm b) 6 cm

c) 9 cm d) 7 cm

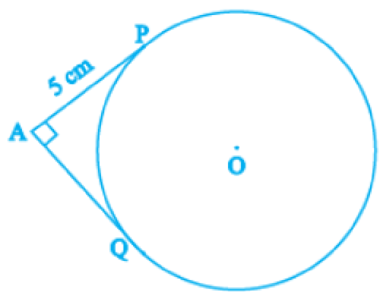
8. We have, $AB||DE$ and $BD||EF$.Then, **[1]**



a) $BC^2 = AB\ .\ CE$ b) $AC^2 = BC\ .\ DC$

c) $AB^2 = AC\ .\ DE$ d) $DC^2 = CF \times AC$

9. The pair of tangents AP and AQ drawn from an external point to a circle with centre O are prependicular to each other and length of each tangent is 5 cm. The radius of the circle is **[1]**



a) 7.5 cm b) 5 cm

c) 10 cm d) 2.5 cm

10. If $2\cos 3\theta = 1$ then θ= ? **[1]**

a) 30° b) 10°

c) 15° d) 20°

11. The angle of elevation of the top of a tower at a point on the ground 50 m away from the foot of the tower is 45°. Then the height of the tower (in metres) is **[1]**

a) $\frac{50}{\sqrt{2}}$ b) $\frac{50}{\sqrt{3}}$

c) $50\sqrt{3}$ d) 50

12. $\frac{2\tan 30^\circ}{1+\tan^2 30^\circ} =$ **[1]**

a) $cos60^\circ$ b) None of these

c) $tan60^\circ$ d) $sin60^\circ$

13. The area of a sector of a circle with a sector angle θ is given by **[1]**

a) $\frac{\pi r\theta}{360}$

b) $\frac{\pi r^2\theta}{360}$

c) $\frac{\pi r^2\theta}{180}$

d) $\frac{2\pi r}{360}$

14. The area of a sector of a circle with radius r, making an angle of x° at the centre is **[1]**

a) $\frac{x}{360} \times \pi r^2$

b) $\frac{x}{100}\ xw^2$

c) $\frac{x}{180} \times 2\pi$

d) $\frac{x}{360} \times 2xr$

15. A game of chance consists of spinning an arrow which comes to rest pointing at one of the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 and these values are equally likely outcomes. The probability that it will point at a number greater than 5 is **[1]**

a) $\frac{1}{2}$

b) $\frac{1}{4}$

c) $\frac{1}{5}$

d) $\frac{1}{3}$

16. In a data, if l = 40, h = 15, f_1 = 7, f_0 = 3, f_2 = 6, then the mode is **[1]**

a) 82

b) 62

c) 52

d) 72

17. A sphere is placed inside a right circular cylinder so as to touch the top, base and lateral surface of the cylinder. If the radius of the sphere is r, then the volume of the cylinder is **[1]**

a) $2\pi r^3$

b) $8\pi r^3$

c) $\frac{8}{3}\pi r^3$

d) $4\pi r^3$

18. The algebraic sum of the deviations of a frequency distribution from its mean is: **[1]**

a) 0

b) a non-zero number

c) always positive

d) always negative

19. **Assertion (A):** If the points A(4, 3) and B(x, 5) lies on a circle with the centre O(2, 3) then the value of x is 2. **[1]**
Reason (R): The mid−point of the line segment joining the points P(x_1, y_1) and Q(x_2, y_2) is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** $\sqrt{a}$ is an irrational number, where a is a prime number. **[1]**
Reason (R): Square root of any prime number is an irrational number.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

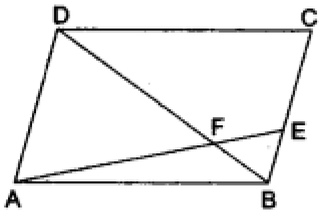
21. Is the pair of linear equation consistent/inconsistent? If consistent, obtain the solution graphically: x – y = 8; 3x – 3y = 16 **[2]**

22. If $\triangle ABC \cong \triangle DEF$, $\angle A = 47^\circ$, $\angle E = 83^\circ$, then find $\angle C$. **[2]**

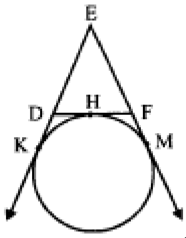
OR

ABCD is a parallelogram and E is a point on BC. If the diagonal BD intersects AE at F, prove that

$AF \times FB = EF \times FD.$



23. In the adjoining figure, a circle touches the side DF of △EDF at H and touches ED and EF produced at K and M respectively. If EK = 9 cm, then what is perimeter of △EDF? **[2]**



24. Prove that: $\frac{\sin\theta}{\cot\theta+\operatorname{cosec}\theta} = 2 + \frac{\sin\theta}{\cot\theta-\operatorname{cosec}\theta}$ **[2]**

25. To warm ships for underwater rocks, a lighthouse spreads a red coloured light over a sector of angle 80° to a distance of 16.5 km. Find the area of the sea over which the ships are warned. (use $\pi = 3.14$) **[2]**

OR

What is the length (in terms of π) of the arc that subtends an angle of 36° at the centre of a circle of radius 5 cm?

Section C

26. Find the greatest number that will divide 445, 572 and 699 leaving remainders 4, 5 and 6 respectively. **[3]**

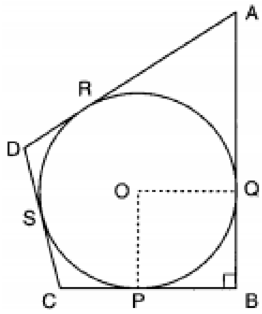
27. Find the zeroes of the quadratic polynomial $7y^2 - \frac{11}{3}y - \frac{2}{3}$ and verify the relationship between the zeroes and the coefficients. **[3]**

28. Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu? Solve the pair of the linear equation obtained by the elimination method. **[3]**

OR

The area of a rectangle remains the same if the length is increased by 7 metres and the breadth is decreased by 3 metres. The area remains unaffected if the length is decreased by 7 metres and breadth is increased by 5 metres. Find the dimensions of the rectangle.

29. In the figure, a circle is inscribed in a quadrilateral ABCD in which ∠B = 90°. If AD = 23 cm, AB = 29 cm and DS = 5 cm, find the radius (r) of the circle. **[3]**



30. In △ABC, right angled at B, AB = 24 cm, BC = 7 cm. Determine: **[3]**
  i. Sin A cos A
  ii. Sin C cos C

OR

A man on the deck of a ship, 12 m above water level, observes that the angle of elevation of the top of a cliff is 60° and the angle of depression of the base of the cliff is 30°. Find the distance of the cliff from the ship and the height of the cliff. [Use $\sqrt{3}$ = 1.732]

31. Two different dice are thrown together. Find the probability that the numbers obtained **[3]**

i. have a sum less than 7

ii. have a product less than 16

iii. is a doublet of odd numbers.

Section D

32. Two pipes running together can fill a tank in $11\frac{1}{9}$ minutes. If one pipe takes 5 minutes more than the other to fill the tank, find the time in which each pipe would fill the tank separately. **[5]**

OR

Represent the situation in the form of the quadratic equation:

A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/hr less, then it would have taken 3 hours more to cover the same distance. We need to find the speed of the train.

33. In ΔABC, AD is the median to BC and in △PQR PM is the median to QR.If $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$. Prove that $\Delta ABC \sim \Delta PQR$. **[5]**

34. A solid is in the shape of a right-circular cone surmounted on a hemisphere, the radius of each of them is being 3.5 cm and the total height of solid is 9.5 cm. Find the volume of the solid. **[5]**

OR

From a cubical piece of wood of side 21 cm, a hemisphere is carved out in such a way that the diameter of the hemisphere is equal to the side of the cubical piece. Find the surface area and volume of the remaining piece.

35. In a retail market, fruit vendors were selling mangoes kept in packing boxes. These boxes contained varying number of mangoes. The following was the distribution of mangoes according to the number of boxes. **[5]**

Number of mangoes	50-52	53-55	56-58	59-61	62-64
Number of boxes	15	110	135	115	25

Find the mean number of mangoes kept in a packing box. Which method of finding the mean did you choose?

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Your friend Varun wants to participate in a 200m race. He can currently run that distance in 51 seconds and with each day of practice it takes him 2 seconds less. He wants to do in 31 seconds.

(i) Write first four terms are in AP for the given situations.

(ii) What is the minimum number of days he needs to practice till his goal is achieved?

OR

How many second takes after 5th days?

(iii) Out of 41, 30, 37 and 39 which term is not in the AP of the above given situation?

37. **Read the text carefully and answer the questions:** **[4]**

Using Cartesian Coordinates we mark a point on a graph by how far along and how far up it is.

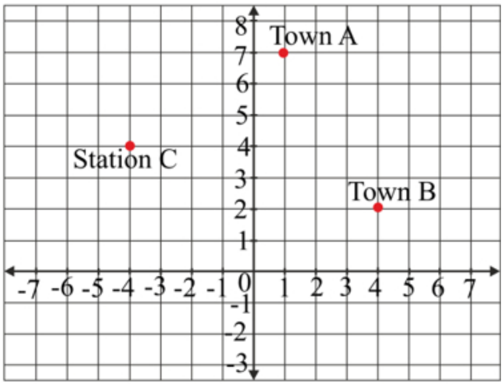
The left-right (horizontal) direction is commonly called X-axis.

The up-down (vertical) direction is commonly called Y-axis.

When we include negative values, the x and y axes divide the space up into 4 pieces.

Read the information given above and below:

Two friends Veena and Arun work in the same office in Delhi. In the Christmas vacations, both decided to go their hometowns represented by Town A and Town B respectively in the figure given below. Town A and Town B are connected by trains from the same station C (in the given figure) in Delhi.



(i) Who will travel more distance to reach their home?

(ii) Find the location of the station.

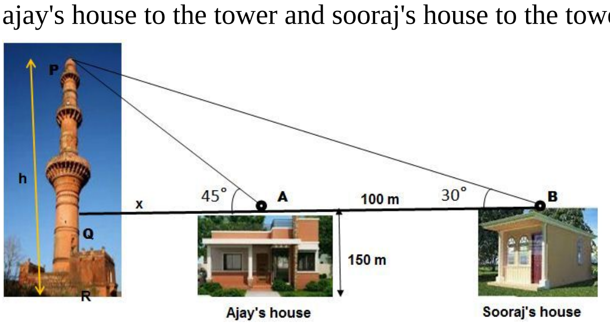
OR

Find the distance between Town A and Town B.

(iii) Find in which ratio Y-axis divide Town B and Station.

38. **Read the text carefully and answer the questions:** **[4]**

The houses of Ajay and Sooraj are at 100 m distance and the height of their houses is the same as approx 150 m. One big tower was situated near their house. Once both friends decided to measure the height of the tower. They measure the angle of elevation of the top of the tower from the roof of their houses. The angle of elevation of ajay's house to the tower and sooraj's house to the tower are 45^o and 30^o respectively as shown in the figure.



(i) Find the height of the tower.

(ii) What is the distance between the tower and the house of Sooraj?

OR

Find the distance between top of the tower and top of Sooraj's house?

(iii) Find the distance between top of tower and top of Ajay's house?

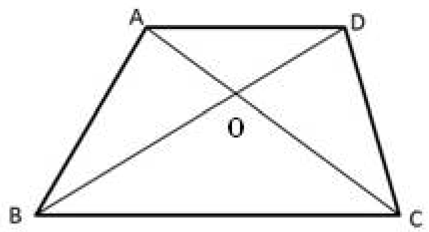
Solution

Section A

1. **(a)** a rational number
   Explanation: (1 + $\sqrt{2}$) + (1 - $\sqrt{2}$) = 1 + $\sqrt{2}$ + 1 - $\sqrt{2}$ = 1 + 1 = 2 And 2 is a rational number.
   Therefore the given number is rational number.
2. **(a)** an even number
   Explanation: Let p_1 and p_2 be 5 two odd primes.
   Then,
   $p_1^2 - p_2^2 = (p_1 - p_2)(p_1 + p_2)$
   We know that sum and difference of two odd numbers is even
   $\therefore$ $(p_1 - p_2)$ *and* $(p_1 + p_2)$ are even numbers.
   Also, we know that product of even numbers is an even number, therefore
   $p_1^2 - p_2^2 = (p_1 - p_2)(p_1 + p_2)$, is an even number.
3. **(a)** -2 < k < 2
   Explanation: For no real roots, we must have: b^2 - 4ac < 0.
   $k^2 - 4 < 0 \Rightarrow k^2 < 4 \Rightarrow -2 < k < 2$.
4. **(b)** $\frac{5}{13}$
   Explanation: Let the fraction be $\frac{x}{y}$.
   According to question
   x + y = 18 ... (i)
   And $\frac{x}{y+2} = \frac{1}{3}$
   $\Rightarrow$ 3x = y + 2
   $\Rightarrow$ 3x - y = 2 ... (ii)
   On solving eq. (i) and eq. (ii), we get
   x = 5, y = 13
   Therefore, the fraction is $\frac{5}{13}$
5. **(a)** 3
   Explanation: Here it is given that y = 1 is a common root, so we have;
   $ay^2 + ay + 3 = 0$
   $\therefore$ a $\times$ $(1)^2$ + a (1) + 3 = 0
   a + a + 3 = 0 $\Rightarrow$ 2a = -3
   $\Rightarrow a = \frac{-3}{2}$
   and $y^2 + y + b = 0$
   $(1)^2$ + (1) + b = 0 $\Rightarrow$ 1 + 1 + b = 0
   $\Rightarrow$ 2 + b = 0
   $\therefore$ b = -2
   ab = $\frac{-3}{2}$ $\times$ (-2) = 3
6. **(c)** $\sqrt{34}$
   Explanation: In rectangle AOBC, AB is a diagonal.
   $\therefore$ AB = $\sqrt{(5-0)^2 + (0-3)^2}$
   = $\sqrt{25+9}$ = $\sqrt{34}$ units

7. **(a)** 8 cm

Explanation:



In ΔAOB and ΔCOD,

∠ AOB = ∠COD (Vertically opposite angles)

$\frac{AO}{OC} = \frac{DO}{OB}$ (Given)

Therefore according to SAS similarity criterion, we have

ΔAOB ~ ΔCOD

$\frac{AO}{OC} = \frac{BO}{OD} = \frac{AB}{DC}$[corresponding sides of similar triangles are proportional]

$\Rightarrow \frac{1}{2} = \frac{4}{DC}$

⇒ DC = 8 cm

8.

(d) $DC^2 = CF \times AC$

Explanation: In $\triangle$ABC, using Thales theorem,

$\frac{\text{DC}}{\text{AC}} = \frac{\text{CE}}{\text{BC}}$ $[AB||DE]$ ....(i)

And in triangle BCD, using Thales theorem,

$\frac{\text{CF}}{\text{DC}} = \frac{\text{CE}}{\text{BC}}$ $[BD||EF]$ ....(ii)

From eq. (i) and (ii), we have

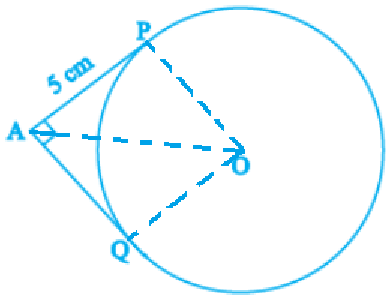
$\frac{\text{DC}}{\text{AC}} = \frac{\text{CF}}{\text{DC}}$

$\Rightarrow DC^2 = CF \times AC$

9.

(b) 5 cm

Explanation:



AP = AQ = 5 cm

(tangent from external point are equal)

Radii makes right angle with tangent

$\triangle$APO ≅ $\triangle$AQO (by R.H.S.)

As $\angle \text{PAQ} = 90^\circ$, $So\ \angle \text{PAO} = 45^\circ$

In $\triangle$APO

$\tan 45^\circ = \frac{\text{OP}}{\text{AP}} = \frac{\text{OP}}{5}$

$\Rightarrow \text{OP} = 5\text{cm}$

Hence, the radii of circle = 5 cm

10.

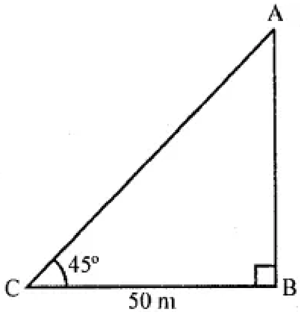
(d) 20°

Explanation: $2\cos 3\theta = 1 \Rightarrow \cos 3\theta = \frac{1}{2} = \cos 60^\circ \quad \Rightarrow 3\theta = 60^\circ \Rightarrow \theta = 20^\circ$

11.

(d) 50

Explanation: Let AB be tower and C is as a point on the ground 52 m away



From foot of tower B

Angle of elevation is 45^{o}

let h be height of tower = x m

$\therefore \tan\theta = \frac{AB}{BC} \Rightarrow$ $\tan 45^{o} = \frac{AB}{5}$

$\Rightarrow 1 = \frac{AB}{50} \Rightarrow = 50$ m

12.

(d) $sin 60^{\circ}$

Explanation: Given: $\frac{2\tan 30^{\circ}}{1+\tan^2 30^{\circ}}$

$= \frac{2\times\frac{1}{\sqrt{3}}}{1+\left(\frac{1}{\sqrt{3}}\right)^2}$

$= \frac{2}{\sqrt{3}\left(\frac{3+1}{3}\right)}$

$= \frac{6}{4\sqrt{3}} = \frac{3}{2\sqrt{3}}$

$= \frac{\sqrt{3}}{2}$

$= \sin 60^{\circ}$

13.

(b) $\frac{\pi r^2\theta}{360}$

Explanation: The area of a sector of a circle with sector angle θ is given by $\frac{\pi r^2\theta}{360^{\circ}}$, where r = radius of the circle

14. **(a)** $\frac{x}{360} \times \pi r^2$

Explanation: Area of a sector of a circle with radius r and making an angle of x^0

at the centre = $\frac{x}{360} \times \pi r^2$

15. **(a)** $\frac{1}{2}$

Explanation: Number of possible outcomes = {6, 7, 8, 9, 10} = 5

Number of total outcomes = 10

$\therefore$ Required Probability = $\frac{5}{10} = \frac{1}{2}$

16.

(c) 52

Explanation: Mode = $l + \left(\frac{f_1-f_0}{2f_1-f_0-f_2}\right) \times h$

= 40 + $\frac{7-3}{7\times2-3-6} \times 15$

= 40 + $\frac{4}{5} \times 15$

= 40 + 12

= 52

17. **(a)** $2\pi r^3$

Explanation: Volume of a sphere = $(4/3)\pi r^3$

Volume of a cylinder = $\pi r^2 h$

Given, sphere is placed inside a right circular cylinder so as to touch the top, base and lateral surface of the cylinder and the radius of the sphere is *r.*

Thus, height of the cylinder = diameter = 2r and base radius = r

Volume of the cylinder = $\pi \times r^2 \times 2r = 2\pi r^3$

18. **(a)** 0

Explanation: The algebraic sum of the deviations of a frequency distribution from its mean is zero.

Let $x_1, x_2, x_3, ...x_n$ are observations and $\overline{X}$ is the mean

$\therefore (\overline{x} - x_1) + (\overline{x} - x_2) + (\overline{x} - x_3) + ...(\overline{x} - xn)$

$= n\overline{x} - (x_1 + x_2 + x_3 + ...x_n)$

$= n\overline{x} - n\overline{x} = 0$

19.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation: We know that the mid-point of the line segment joining the points

$\boldsymbol{P}(\boldsymbol{x}_1, \boldsymbol{y}_1)$ and $\boldsymbol{Q}(\boldsymbol{x}_2, \boldsymbol{y}_2)$ is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

So, the Reason is correct.

Given, the points A(4, 3) and B(x, 5) lie on a circle with center O(2, 3).

Then $OA = OB \Rightarrow (OA)^2 = (OB)^2$

$\Rightarrow (4-2)^2 + (3-3)^2 = (x-2)^2 + (5-3)^2$

$\Rightarrow (2)^2+(0)^2 = (x-2)^2 + (2)^2 \Rightarrow 4 = (x-2)^2 + 4 \Rightarrow (x-2)^2 = 0$

$\Rightarrow x - 2 = 0 \Rightarrow x = 2$

So Assertion is correct.

The correct option is Both A and R are true but R is not the correct explanation of A.

20. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: As we know that square root of every prime number is an irrational number. So, both assertion and reason are correct and reason explains assertion.

Section B

21. x - y = 8.................(1)

3 x - 3 y = 16.............(2)

Here, $a_1 = 1, b_1 = -1, c_1 = -8$

$a_2 = 3, b_2 = -3, c_2 = -16$

We see that $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, the lines represented by the equations(1) and (2) are parallel.

Therefore, equations (1) and (2) have no solution, i.e., the given pair of linear equation is inconsistent.

22. Given: $\triangle ABC \cong \triangle DEF$

$\Rightarrow \angle A = \angle D, \angle B = \angle E, \angle C = \angle F$

Also $\angle A = 47^o, \angle E = 83^o = \angle B$

By angle sum property for triangles,

$\angle A + \angle B + \angle C = 180^o$

$47^o + 83^o + \angle C = 180^o$

$130^o + \angle C = 180^o$

$\angle C = 180^o - 130^o$

$\angle C = 50^o$

OR

Given: ABCD is a parallelogram and E is a point on BC. The diagonal BD intersects AE at F.

To prove: $AF \times FB = EF \times FD$

Proof: Since ABCD is a parallelogram, then its opposite sides must be parallel.

$\therefore$In $\triangle ADF$ and $\triangle EBF$

$\angle FDA = \angle EBF$ and $\angle FAD = \angle FEB$ [Alternate interior angles]

$\angle AFD = \angle BFE$ [vertically opposite angles]

Therefore,by AAA criteria of similar triangles,we have,

$\triangle ADF = \triangle EBF$

Since the corresponding sides of similar triangles are proportional. Therefore,we have,

$\frac{AF}{FD} = \frac{EF}{FB}$
$\Rightarrow AF \times FB = EF \times FD$

23. We know that tangent segments to a circle from the same external point are Equal. Therefore, we have

EK = EM = 9 cm

Now, EK + EM = 18 cm

$\Rightarrow ED + DK + EF + FM = 18cm$

$\Rightarrow ED + DH + EF + HF = 18cm$

$\Rightarrow ED + DF + EF = 18cm$

$\Rightarrow$ Perimeter of $\triangle$EDF $= 18cm$

24. We have,

$\frac{\sin\theta}{\cot\theta+\cos ec\theta} = 2 + \frac{\sin\theta}{\cot\theta-\cos ec\theta}$ or, $\frac{\sin\theta}{\cot\theta+\cos ec\theta} - \frac{\sin\theta}{\cot\theta-\cos ec\theta} = 2$

Now,

LHS $= \frac{\sin\theta}{\cot\theta+\cos ec\theta} - \frac{\sin\theta}{\cot\theta-\cos ec\theta}$

$\Rightarrow \quad \text{LHS} = \frac{\sin\theta}{\cos ec\theta+\cot\theta} + \frac{\sin\theta}{\cos ec\theta-\cot\theta}$

$\Rightarrow \quad \text{LHS} = \sin\theta\left\{\frac{1}{\cos ec\theta+\cot\theta} + \frac{1}{\cos ec\theta-\cot\theta}\right\}$

$\Rightarrow \quad \text{LHS} = \sin\theta\left\{\frac{\cos ec\theta-\cot\theta+\cos ec\theta+\cot\theta}{\cos ec^2\theta-\cot^2\theta}\right\} = \sin\theta\left(\frac{2\cos ec\theta}{1}\right)$

$\Rightarrow \quad \text{LHS} = \sin\theta(2\cos ec\theta) = 2\sin\theta \times \frac{1}{\sin\theta} = 2 = \text{RHS}$

25. We have, r = 16.5 km and θ = 80°.

Let A be the area of the sea over which the ships are warmed. Then,

$A = \frac{\theta}{360} \times \pi r^2 = \frac{80}{360} \times 3.14 \times 16.5 \times 16.5\ km^2 = 189.97\ km^2$

OR

We have

r = 5 cm

$\theta = 36^\circ$

We have to find the length of the arc.

Length of the arc$= \frac{\theta}{360} \times 2\pi r$

Substituting the values we get,

Length of the arc $= \frac{36}{360} \times 2\pi \times 5$ ...(1)

Now we will simplify the equation (1) as below,

$= \frac{1}{10} \times 2\pi \times 5$

$= \frac{1}{2} \times 2\pi$

= π

Therefore, the length of the arc is π cm.

Section C

26. We have to find the greatest number that divides 445, 572 and 699 and leaves remainders of 4, 5 and 6 respectively. This means when the number divides 445, 572 and 699, it leaves remainders 4, 5 and 6. It means that

445 - 4 = 441,

572 - 5 = 567

and 699 - 6 = 693

are completely divisible by the required number.

For the highest number which divides the above numbers we need to calculate HCF of 441, 567 and 693 .

Therefore, the required number is the H.C.F. of 441, 567 and 693 Respectively.

First, consider 441 and 567.

By applying Euclid's division lemma, we get

567 = 441 $\times$ 1 + 126

441 = 126 $\times$ 3 + 63

126 = 63 $\times$ 2 + 0.

Therefore, H.C.F. of 441 and 567 = 63

Now, consider 63 and 693

again we have to apply Euclid's division lemma, we get

693 = 63 $\times$ 11 + 0.

Therefore, H.C.F. of 441, 567 and 693 is 63

Hence, the required number is 63. 63 is the highest number which divides 445,572 and 699 will leave 4,5 and 6 as remainder respectively.

27. $p(y) = 7y^2 - \frac{11}{3}y - \frac{2}{3} = \frac{1}{3}(21y^2 - 11y - 2)$

$= \frac{1}{3}(21y^2 - 14y + 3y - 2)$

$= \frac{1}{3}[7y(3y - 2) + 1(3y - 2)]$

$= \frac{1}{3}[(7y + 1)(3y - 2)]$

$\therefore$ Zeroes are $\frac{2}{3}, -\frac{1}{7}$

Sum of Zeroes = $\frac{2}{3} - \frac{1}{7} = \frac{11}{21}$

$\frac{-b}{a} = \frac{11}{21}$

$\therefore$ sum of zeroes = $\frac{-b}{a}$

Product of Zeroes = $(\frac{2}{3})(-\frac{-1}{7}) = -\frac{2}{21}$

$\frac{c}{a} = -\frac{2}{3}(\frac{1}{7}) = -\frac{2}{21}$

$\therefore$ Product = $\frac{c}{a}$

28. Let the present age of Nuri and Sonu be x years and y years respectively.

Then, according to the question,

$x - 5 = 3(y - 5)$

$\Rightarrow x - 5 = 3y - 15$

$\Rightarrow x - 3y = -10$ .... (1)

$x + 10 = 2(y + 10)$

$x + 10 = 2y + 20$

$\Rightarrow x - 2y = 10$ ------------ (2)

Subtracting equation (2) from equation (1), we get

$-y = -20$

$\Rightarrow y = 20$

Subtracting equation (2) from equation (1), we get

$x - 2(20) = 10$

$\Rightarrow x - 40 = 10$

$\Rightarrow x = 40 + 10$

$\Rightarrow x = 50$

Hence, Nuri and Sonu are 50 years and 20 years old respectively at present.

Verification.Subtracting the value of x = 50 and y = 20,we find that both the eqations (1) and (2) are satisfied as shown below:

$x - 3y = 50 - 3(20) = 50 - 60 = -10$

$x - 2y = 50 - 2(20) = 50 - 40 = 10$

Hence,the solution is correct.

OR

Let us suppose that the length and breadth of the rectangle be x m and y m respectively.

Then, Area of rectangle = xy meter^2

Now, according to question if length is increased by 7m and the breadth is decreased by 3m, the area remains same

$\therefore xy = (x + 7)(y - 3)$

$\Rightarrow xy = xy - 3x + 7y - 21$

$\Rightarrow 3x - 7y = -21$ ....(i)

Again, according to question when length is decreased by 7m and breadth is increased by 5m, then area remains unaffected

$\therefore xy = (x - 7)(y + 5)$

$\Rightarrow xy = xy + 5x - 7y - 35$

$\Rightarrow 35 = 5x - 7y$

$\Rightarrow 5x - 7y = 35$ ....(ii)

Subtracting equation (i) from (ii), we get

$5x - 7y -(3x- 7y) = 35 - (-21)$

or, $5x - 7y -3- + 7y = 35 + 21$

$\Rightarrow 2x = 56$

$\Rightarrow x = \frac{56}{2} = 28$

Put the value of x = 28 in equation (ii), we get

$5 \times 28 - 7y = 35$

$\Rightarrow 140 - 7y = 35$

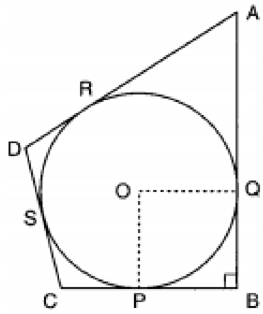
$\Rightarrow -7y = 35 - 140$

$\Rightarrow -7y = -105$

$\Rightarrow y = \frac{105}{7} = 15$

Therefore, dimensions of the rectangle are 28m and 15m respectively.

29. Given,a circle is inscribed in a quadrilateral ABCD



$OQ \perp AB$ [Radius is perpendicular to the tangent]

and $OP \perp BC$

OQ = OP [Radii of a circle]

$\therefore$ OPBQ is a square.

$\Rightarrow BQ = BP = OP = r.$

Now, $RD = DS$

$\Rightarrow RD = 5\ cm$

$\therefore AR = AD - RD$

$= 23 - 5$

$= 18\ cm$

Also, $AR = AQ$

$\Rightarrow AQ = 18\ cm$

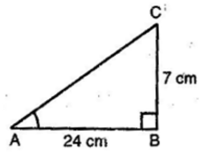
Now, $AB = AQ + BQ$

$\Rightarrow 29 = 18 + r$

$\Rightarrow r = 11\ cm.$

30. Let us draw a right angled triangle ABC, right angled at B.

Using Pythagoras theorem,



Given, AB = 24cm and BC = 7cm

Using Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$AC^2 = 24^2 + 7^2 = 576 + 49 = 625$

$\therefore$ AC = 25 cm

i. $\sin A = \frac{P}{H} = \frac{BC}{AC} = \frac{7}{25}$, $\cos A = \frac{B}{H} = \frac{AB}{AC} = \frac{24}{25}$

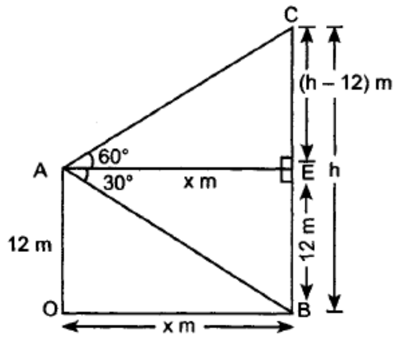
$\Rightarrow sinA.cosA = \frac{7}{25} \times \frac{24}{25} = \frac{168}{625}$

ii. $\sin C = \frac{P}{H} = \frac{AB}{AC} = \frac{24}{25}$, $\cos C = \frac{B}{H} = \frac{BC}{AC} = \frac{7}{25}$

$\Rightarrow sinC.cosC = \frac{24}{25} \times \frac{7}{25} = \frac{168}{625}$

OR

A is the position of the man, OA = 12 m, BC is cliff.



BC = h m and CE = (h - 12)m

Let $\mathrm{AE}=\mathrm{OB}=x \mathrm{~m}$

In right angled triangle AEB,

$\frac{\mathrm{AE}}{\mathrm{BE}}=\cot 30^{\circ} \Rightarrow \mathrm{AE}=12 \times \sqrt{3}$

$=12 \times 1.732 \mathrm{m}=20.78 \mathrm{m}$

$\therefore$ Distance of ship from cliff = 20.78 m.

In right angled triangle AEC,

$\frac{C E}{A E}=\tan 60^{\circ} \Rightarrow \frac{h-12}{12 \sqrt{3}}=\sqrt{3}$

$h-12=36 \Rightarrow h=48 \mathrm{m}$

31. When 2 dice are rolled,

The possible outcomes are :

(1,1),(1,2),(1,3),(1,4),(1,5),(1,6)

(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)

(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)

(4,1),(4,2),(4,3),(3,4),(3,5),(3,6)

(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)

(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)

$\therefore$Total number of outcomes = 36

i. Let A be the event of getting the numbers whose sum is less than 7.

Number of favourable outcomes =15

Favourable outcomes are (1,1),(1,2),(1,3),(1,4),(1,5),(2,1),(2,2),(2,3),(2,4),(3,1),(3,2),(3,3),(4,1),(4,2) and (5,1).

$\therefore P(A)=\frac{\text { Number of favourable outcomes }}{\text { Total number of outcomes }}=\frac{15}{36}=\frac{5}{12}$

ii. Let B be the event of getting the numbers whose product is less than 16.

Number of favourable outcomes= 25

Favourable outcomes are (1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),(3,1),(3,2),(3,3),(3,4),(3,5),(4,1),(4,2),(4,3),(5,1),(5,2),(5,3),(6,1) and (6,2).

$\therefore P(A)=\frac{\text { Number of favourable outcomes }}{\text { Total number of outcomes }}=\frac{25}{36}$

iii. Let C be the event of getting the numbers which are doublets of odd numbers.

Number of favourable outcomes=3

Favourable outcomes are (1,1),(3,3) and (5,5).

$\therefore P(A)=\frac{\text { Number of favourable outcomes }}{\text { Total number of outcomes }}=\frac{3}{36}=\frac{1}{12}$

Section D

32. Let time taken by pipe A be x minutes, and time taken by pipe B be x + 5 minutes.

In one minute pipe A will fill $\frac{1}{x}$ tank

In one minute pipe B will fill $\frac{1}{x+5}$ tank

pipes A + B will fill in one minute = $\frac{1}{x}+\frac{1}{x+5}$ tank

Now according to the question.

$\frac{1}{x}+\frac{1}{x+5}=\frac{9}{100}$

or, $\frac{x+5+x}{x(x+5)}=\frac{9}{100}$

or, 100(2x + 5) = 9x(x + 5)

or, $200 x+500=9 x^{2}+45 x$

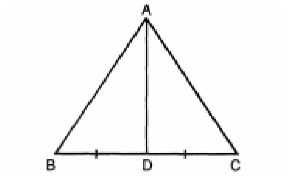
or, $9 x^{2}-155 x-500=0$

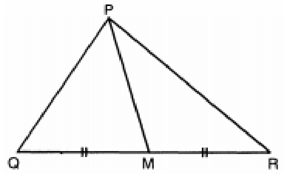
or, $9x^2 - 180x + 25x - 500 = 0$
or,9x(x - 20) + 25(x - 20) = 0
or, (x-20)(9x + 25) = 0
or, $x = 20, \frac{-25}{9}$
rejecting negative value, x = 20 minutes
and x + 5 = 25 minutes
Hence pipe A will fill the tank in 20 minutes and pipe B will fill it in 25 minutes.

OR

Distance travelled by the train = 480 km
Let the speed of the train be x kmph
Time taken for the journey = $\frac{480}{x}$
Given speed is decreased by 8 kmph
Hence the new speed of train = (x – 8) kmph
Time taken for the journey = $\frac{480}{x-8}$
$\frac{480}{x-8} = \frac{480}{x} + 3$
$\Rightarrow \frac{480}{x-8} - \frac{480}{x} = 3$
$\Rightarrow \frac{480(x-x+8)}{x(x-8)} = 3$
$\Rightarrow \frac{480\times 8}{x(x-8)} = 3$
$\Rightarrow 3x(x-8) = 480 \times 8$
$\Rightarrow x(x-8) = 160 \times 8$
$\Rightarrow x^2 - 8x - 1280 = 0$

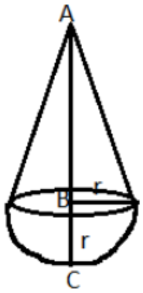
33. Proof: In ΔABC $BC = 2BD$





and In ΔPQR $QR = 2QM$
Given, $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$
or, $\frac{AB}{PQ} = \frac{AD}{PM} = \frac{BC}{QR} = \frac{2BD}{2QM} = \frac{BD}{QM}$
So, in ΔABD and ΔPQM
$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{BD}{QM}$
$\therefore \Delta ABD \sim \Delta PQM$
So $\angle B = \angle Q$ (By CPCT)
In ΔABC and ΔPQR
$\angle B = \angle Q$
and $\frac{AB}{PQ} = \frac{BC}{QR}$
Hence $\Delta ABC \sim \Delta PQR$

34.



From the given figure,

Height (AB) of the cone = AC - BC (Radius of the hemisphere)
Thus, height of the cone = Total height - Radius of the hemisphere
= 9.5 - 3.5
= 6 cm
Volume of the solid = Volume of the cone + Volume of the hemisphere
$= \left(\frac{1}{3}\pi r^2 h\right) + \left(\frac{2}{3}\pi r^3\right)$
$= \frac{1}{3}\pi r^2(h + 2r)$
$= \frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5(6 + 2 \times 3.5)$
$= \frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 13$
= 166.83 cm^3
Thus, total volume of the solid is 166.83 cm^3.

OR

Given side of a cube = 21 cm
Diameter of the hemisphere is equal to the side of the cubical piece (d) = 21 cm
$\Rightarrow$ Radius of the hemisphere = 10.5 cm
Volume of cube = $Side^3$
= $(21)^3$
= 9261 cm^3
Surface area of cubical piece of wood = $6a^2$
= 6 × 21 × 21 cm^2
= 2646 cm^2
Volume of the hemisphere $= \frac{2}{3}\pi r^3$
$= \frac{2}{3} \times \frac{22}{7} \times 10.5 \times 10.5 \times 10.5$
= 44 × 0.5 × 10.5 × 10.5
= 2425.5 cm^3
Surface area of hemisphere = $2\pi r^2$
= 2 × π × 10.5 × 10.5 cm
= 693 cm
Volume of remaining solid = Volume of cubical piece of wood – Volume of hemisphere
$\Rightarrow$ Volume of the remaining solid = 9261 - 2425.5
= 6835.5 cm^3
Surface area remaining piece of solid = surface area of cubical piece of wood – Area of circular base of hemisphere + Curved Surface area of hemisphere
= $6a^2 - \pi r^2 + 2\pi r^2$
= $(2646 - \pi \times 10.5^2 + 693)$ cm^2
= 2992.5 cm^2.

35. Since value of number of mangoes and number of boxes are large numerically. So we use step-deviation method

True Class Interval	**No. of boxes(f_i)**	**Class mark(x_i)**	$u_i = \frac{x_i - a}{h}$	f_iu_i
49.5-52.5	15	51	-2	-30
52.5-55.5	110	54	-1	-110
55.5-58.5	135	57	0	0
58.5-61.5	115	60	1	115
61.5-64.5	25	63	2	50
	$\sum f_i$ = 400			$\sum f_iu_i$ = 25

Let assumed mean (a) = 57,
h = 3 ,

$\therefore \overline{u} = \frac{\sum f_i u_i}{\sum f_i} = \frac{25}{400} = 0.0625$ (approx.)

Using formula, Mean $(\overline{x})$ = a + h$\overline{u}$

= 57 + 3 (0.0625

= 57 + 0.1875

= 57.1875

= 57.19 (approx)

Therefore, the mean number of mangoes is 57.19

Section E

36. **Read the text carefully and answer the questions:**

Your friend Varun wants to participate in a 200m race. He can currently run that distance in 51 seconds and with each day of practice it takes him 2 seconds less. He wants to do in 31 seconds.

(i) 51, 49, 47, ... 31 AP

d = -2

First 4 terms of AP are: 51, 49, 47, 45 ...

(ii) 51, 49, 47, ... 31 AP

d = -2

t_n = a +(n - 1)d

31 = 51 + (n - 1) (-2)

31 = 51 - 2n + 2

31 = 53 - 2n

31 - 53 = -2n

-22 = -2n

n = 11

i.e., he acheived his goal in 11 days.

OR

51, 49, 47, ... 31 AP

d = -2

t_6 = a + (n - 1)d

= 51 + (6 - 1)(-2)

= 51 + (-10)

= 41 sec

(iii)The given AP is

51, 49, 47, 45, 43, 41, 39, 37, 35, 33, 31, 29

$\therefore$ 30 is not in the AP.

37. **Read the text carefully and answer the questions:**

Using Cartesian Coordinates we mark a point on a graph by how far along and how far up it is.

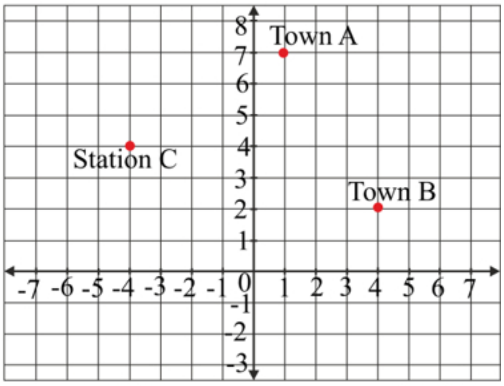
The left-right (horizontal) direction is commonly called X-axis.

The up-down (vertical) direction is commonly called Y-axis.

When we include negative values, the x and y axes divide the space up into 4 pieces.

Read the information given above and below:

Two friends Veena and Arun work in the same office in Delhi. In the Christmas vacations, both decided to go their hometowns represented by Town A and Town B respectively in the figure given below. Town A and Town B are connected by trains from the same station C (in the given figure) in Delhi.



(i) 

Distance travelled by veena = $\sqrt{1-(-4)^2+(7-4)^2}$

$=\sqrt{5^2+3^2}$

$=\sqrt{25+9}$

$=\sqrt{34}$

Distance travelled by Arun = $\sqrt{(4-(-4))^2+(2-4)^2}$

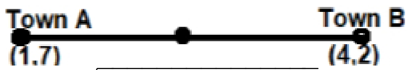
$=\sqrt{64+4}$

$=\sqrt{68}$

$\therefore$ Arun will travel more distance to reach his home.

(ii) Location of station = (-4, 4)

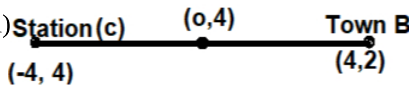
OR



AB = $\sqrt{(4-1)^2+(2-7)^2}$

$=\sqrt{9+25}$

$=\sqrt{34}$

(iii) 

Let y-axis divides station (c) and Town B in K : 1

$0 = \frac{4k-4}{k+1}$

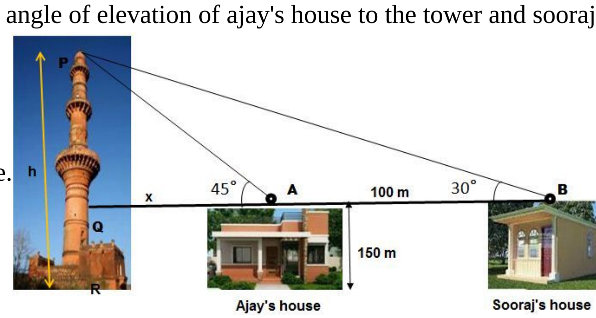
4k = 4

k = 1

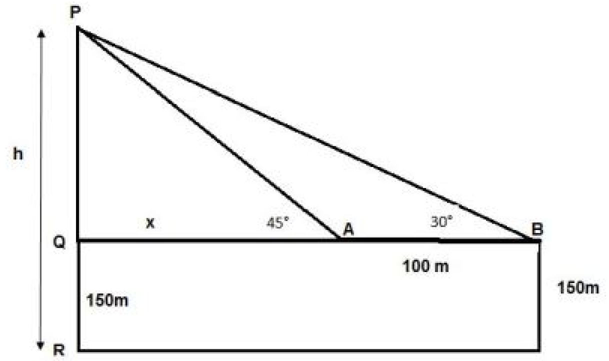
$\therefore$ y-axis divides in 1 : 1

38. **Read the text carefully and answer the questions:**

The houses of Ajay and Sooraj are at 100 m distance and the height of their houses is the same as approx 150 m. One big tower was situated near their house. Once both friends decided to measure the height of the tower. They measure the angle of elevation of the top of the tower from the roof of their houses. The angle of elevation of ajay's house to the tower and sooraj's house to the tower are 45^o and 30^o respectively as shown in the figure.



(i) The above figure can be redrawn as shown below:



Let PQ = y

In △PQA,

tan 45 = $\frac{PQ}{QA} = \frac{y}{x}$

$1 = \frac{y}{x}$

x = y ...(i)

In △PQB,

tan 30 = $\frac{PQ}{QB} = \frac{PQ}{x+100} = \frac{y}{x+100} = \frac{x}{x+100}$

$\frac{1}{\sqrt{3}} = \frac{x}{x+100}$

$x\sqrt{3}$ = x + 100

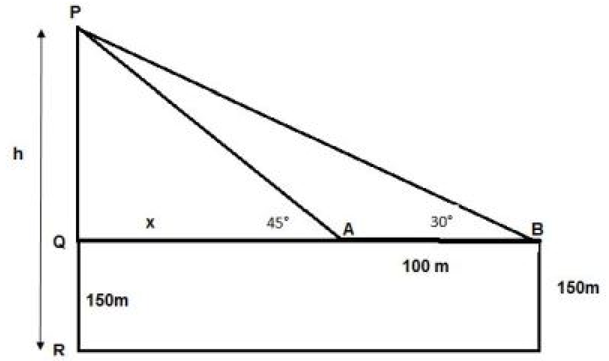
x = $\frac{100}{\sqrt{3}-1}$ = 136.61 m

From the figure, height of tower h = PQ + QR

= x + 150 = 136.61 + 150

h = 286.61 m

(ii) The above figure can be redrawn as shown below:

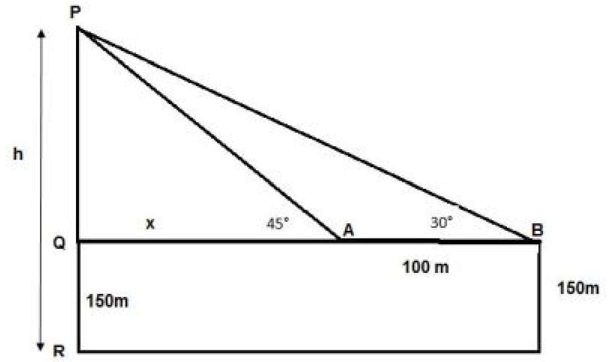


Distance of Sooraj's house from tower = QA + AB

= x + 100 = 136.61 + 100 = 236.61 m

OR

The above figure can be redrawn as shown below:



Distance between top of tower and Top of Sooraj's house is PB

In △PQB

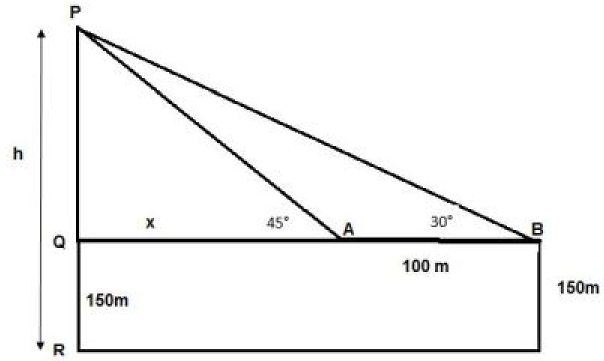
sin 30^o = $\frac{PQ}{PB}$

$\Rightarrow PB = \frac{PQ}{\sin 30^o}$

$\Rightarrow$ PB = $\frac{y}{\frac{1}{2}} = 2 \times 136.61$

$\Rightarrow$ PB = 273.20 m

(iii)The above figure can be redrawn as shown below:



Distance between top of the tower and top of Ajay's house is PA

In $\triangle$PQA

sin 45^o = $\frac{PQ}{PA}$

$\Rightarrow PA = \frac{PQ}{\sin 45^o}$

$\Rightarrow$ PA = $\frac{y}{\frac{1}{\sqrt{2}}} = \sqrt{2} \times 136.61$

$\Rightarrow$ PA = 193.20 m